

# Zeeman and Orbital Limiting Fields: Separated Spin and Charge Degrees of Freedom in Cuprate Superconductors

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Recent in-plane thermal (Nernst) and interlayer (tunnelling) transport experiments in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}$  high temperature superconductors report hugely different limiting magnetic fields. Based on pairing (and the uncertainty principle) combined with the definitions of the Zeeman energy and the magnetic length, we show that in the underdoped regime both fields convert to the same (normal state) pseudogap energy scale  $T^*$  upon transformation as orbital and spin (Zeeman) critical fields, respectively. We reconcile these seemingly disparate findings invoking separated spin and charge degrees of freedom residing in different regions of a truncated Fermi surface.

Establishing the fundamental length and energy scales associated with the superconducting state in the cuprates is pivotal to understanding the origin of the high transition temperature  $T_c$ . The values of the upper critical field  $H_{c2}$  are of particular importance, for they mark the onset of superconducting correlations and directly inform on microscopic parameters such as the coherence lengths in the superconducting state. Early attempts [1,2] have demonstrated the difficulty in mapping the  $H_{c2}$  boundary: it lies in part in the high field range required, but more fundamentally in the large thermal fluctuation regime [3] and the loss of long-range phase coherence below  $H_{c2}(T)$ , making this limiting field ‘fuzzy’ and hard to pinpoint using the usual experimental tools, e.g., transport or magnetization. Complicating matters further is the normal state pseudogap [4] which dominates the phase diagram [5] and whose still unresolved connection to superconductivity is central to the issue of the onset of pairing and coherence. In this context, the stark difference in the doping dependencies of the pseudogap energy scale  $T^*$  and  $T_c$  has been well established:  $T^*$  is decreasing (linearly) with charge doping, while  $T_c$  follows the well known superconducting ‘dome’ described by the phenomenological formula [6]  $T_c/T_c^{\max} = 1 - 82.6(p - 0.16)^2$ .

In a view where pairing correlations onset at  $T^*$  and then acquire global coherence at a lower energy scale  $T_c$ , the region  $T_c \leq T \leq T^*$  is a vast fluctuation regime. The question remains about the doping dependencies of the relevant magnetic field scales, the field  $H_{c2}$  limiting the regime of superconducting response and the pseudogap closing field  $H_{pg}$ . Here we demonstrate the *interconnectedness* of three magnetic scales: (i) the Zeeman field corresponding to the onset of spin correlations at the pseudogap scale  $T^*$  and scaling linear with  $T^*$ ; this field is identified with the experimentally observed [5] pseudogap closing field  $H_{pg}$ , (ii) the orbital critical field  $H_{c2}^*$  quadratic in  $T^*$ , corresponding to the onset of charge correlations and experimentally determined ( $\rightarrow H_{c2}^N$ ) via thermal (Nernst) transport measurements [7], and (iii) the upper critical field  $H_{c2}$  that marks the

onset of global superconducting coherence and which has been experimentally tracked ( $\rightarrow H_{sc}$ ) through the presence of large interplane Josephson currents [10]. This last field,  $H_{sc} \sim H_{c2}$ , follows the superconducting ‘dome’ according to  $H_{sc}(p) \sim 1.4T_c(p)$  and coincides with the usual unique upper critical field  $H_{c2}$  on the strongly overdoped side of the dome. As we will present below, the distinctly different orbital ( $H_{c2}^* \sim H_{c2}^N$ ) and the Zeeman ( $H_{pg}$ ) limiting fields can coexist owing to charge and spin degrees of freedom separated to different parts of the cuprates’ strongly anisotropic Fermi surface.

Before discussing their interrelation, we first briefly recapitulate the origin of the three magnetic field scales  $H_{sc}$ ,  $H_{c2}^N$ , and  $H_{pg}$ . In cuprates, the conventional derivation of the coherence length  $\xi$  through an evaluation of  $H_{c2} = \Phi_0/2\pi\xi^2$  from transport measurements [8,9] ( $\Phi_0 = hc/2e$  denotes the flux quantum) has proved notoriously unreliable: a consequence of the presence of a large vortex liquid regime [3] and the lack of sharp features in the resistivity. A feature that *can* be accurately mapped from the field dependence of the interlayer  $c$ -axis resistivity  $\rho_c(H) = \sigma_c^{-1}(H)$  is the peak at  $H_{sc}$  that corresponds to a crossover from the mostly Josephson (Cooper pair) tunnelling conductivity  $\sigma_J(H)$  at low fields to that of quasiparticles,  $\sigma_q(H)$ , at fields above  $H_{sc}$  [10]. From the measurements of  $\rho_c(H)$  in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}$  [5,10,11] we have found that for *all* doping levels  $H_{sc}(T)$  is nearly  $T$ -exponential (see inset in Fig. 1).

To understand the temperature dependence of  $H_{sc}$  and its connection to the coherence length  $\xi$  we explicitly write the experimentally established form for the  $c$ -axis conductivity at high fields [10]

$$\sigma_c \simeq \underbrace{\sigma_{J0} \exp \left[ \frac{U(H)}{T} \right]}_{\sigma_J} + \underbrace{\sigma_{q0} \left( 1 + \frac{H}{H_{c2}^*} \right)}_{\sigma_q}; \quad (1)$$

the two-channel tunnelling process comprises a term  $\sigma_J$  controlled by the thermally activated diffusive drift of pancake vortices hopping over the energy barriers

$U(H)$  in the  $\text{CuO}_2$  planes [12] and a term  $\sigma_q$  due to (nodal) quasiparticle tunnelling [10]. Here,  $\sigma_{J0}$  and  $\sigma_{q0}$  are the corresponding  $T \rightarrow 0$  extrapolations and  $H_{c2}^* \sim \Phi_0 T^{*2} / \hbar^2 v_F^2$ , where  $T^*$  coincides with the gap in the quasiparticle spectrum [10]. Taking the derivative of Eq. (1) with respect to  $H$  and recalling that  $U(H) \sim U_0 \ln(a_0/\xi) \propto \ln(H_{c2}/H)$  for a 2D vortex lattice [13,14], we obtain the expression for the field  $H_{sc}$  where the maximum in  $\rho_c$  (minimum in  $\sigma_c$ ) will occur,

$$H_{sc}(T) \approx H_{c2} \left[ \frac{\sigma_{q0}}{\sigma_{J0}} \frac{T}{U_0} \frac{H_{c2}}{H_{c2}^*} \right]^{-T/U_0}. \quad (2)$$

Indeed, at low temperatures this high-field  $H_{sc}(T)$  can hardly be distinguished in the experiments from a  $T$ -exponential behavior [15] [for  $x \rightarrow 0$ ,  $x^{-x} = \exp(-x \ln x) \rightarrow 1$ ] displayed in the inset of Fig. 1. Hence, in the zero temperature limit  $H_{sc}|_{T \rightarrow 0} \rightarrow H_{c2}(0)$ . Experimentally, we find the zero temperature values of  $H_{sc}$  as a function of hole doping to follow a parabolic dependence proportional to  $T_c(p)$ , defining the ‘coherence dome’ shown in Fig. 1. Consistent with this, a corresponding ‘dome’-region in the field-doping ( $H - p$ ) phase space has been recently mapped [16] from the systematic doping dependence of the coherence length (the size of the vortex core) using detailed magnetization measurements in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ .

Let us now consider the field scales corresponding to the pseudogap energy  $T^*$ . Recently, Wang *et al.* [7] deduced values of an orbital limiting ‘upper critical field’  $H_{c2}^N$ , past which the charge pairing amplitude should vanish. The  $H_{c2}^N$  was extracted [17] from a linear extrapolation (to zero) [18] of a remarkably large Nernst signal (attributed to vortex-like excitations surviving beyond the point where superconducting phase coherence has been established [3]) and from scaling arguments [7].  $H_{c2}^N$  was found to decrease steeply with increased doping, implying that the Cooper pairing potential and the superfluid density follow opposite trends versus charge doping. This led Wang *et al.* to a radical interpretation [7] of the role of phase fluctuations in the low doping region. Central to understanding this observation is how the Nernst-derived  $H_{c2}^N$  relates to the gap  $T^*$  observed by angle-resolved photoemission (ARPES) [19–21], as well as by the intrinsic tunnelling [22] spectroscopies: pairing correlations are quenched through localization in a magnetic field  $H$  once the magnetic length  $a_0 = \sqrt{\Phi_0/H}$  drops below the pair correlation length  $\xi^* = \hbar v_F / \alpha T^*$ . Here,  $v_F$  is the Fermi velocity and  $\alpha$  is a numerical of order unity [23]. Indeed, the Nernst-derived magnetic field appears to well match this condition [7] and hence qualifies as an orbital limiting (or critical) magnetic field  $H_{c2}^N \sim H_{c2}^*$ ; as such it scales *quadratically* in  $T^*$ ,  $\mu_B H_{c2}^N \sim T^{*2} / m v_F^2$ , where  $\mu_B$  is the Bohr magneton. Note that within the frame of BCS theory  $H_{c2}^*$  corresponds to the conventional  $H_{c2} = \Phi_0 / 2\pi \xi^2 \sim \Phi_0 T_c^2 / \hbar^2 v_F^2$ . In the cuprates, how-

ever,  $T^*$  and  $T_c$  are taken to define separate length ( $\xi^*$  and  $\xi$ ) and field scales ( $H_{c2}^*$  and  $H_{c2}$ ) associated with the appearance of local charge correlations and global phase coherence, respectively (see [24] for a discussion of  $H_{c2}$  in underdoped cuprates; the appearance of two length/field scales has also been discussed within a BCS-Bose Einstein crossover scenario [25]).

Remarkably, an even higher critical magnetic field  $H_{pg} > H_{c2}^N$  has been derived from  $c$ -axis interlayer tunneling transport  $\rho_c$  [5]. In these experiments, the recovery of the normal (ungapped) state  $c$ -axis conductivity indicates that the pseudogap  $T^*$  closes at a much larger field scale  $H_{pg}$  (nearly twice at low doping, see Fig. 2). Again, this limiting field relates to the pseudogap energy scale  $T^*$ , this time, however, via the *linear* Zeeman relation  $k_B T^* = g \mu_B H_{pg}$ , where  $g \sim 2$  [26] denotes the (spectroscopic splitting) Landé  $g$ -factor of the  $\text{Cu}^{2+}$  ions. Correspondingly, one argues that spin-singlets are unpaired at the pseudogap closing field  $H_{pg}$ . Indeed, the field  $H_{pg}$  relates to  $T^*$  via the linear Zeeman scaling irrespective of whether the applied field is across  $\text{CuO}_2$  planes or in-plane [27]. The observed field anisotropy [28] is only that of the  $g$ -factor [29], strengthening the view that the pseudogap is of spin-singlet origin.

Given the equivalence of the limiting fields  $H_{pg}$  and  $H_{c2}^N$  to the same pseudogap energy scale  $T^*$  but via different routes, ‘orbital’ for  $H_{c2}^N$  and ‘Zeeman’ for  $H_{pg}$ , we can simply derive how the two fields relate (as a function of doping  $p$ ),

$$H_{c2}^N(p) = H_{c2}^*(p) \equiv \alpha^2 \frac{\mu_B H_{pg}(p)}{m v_F^2} H_{pg}(p). \quad (3)$$

Note that Eq. (3) rests only on pairing (and the uncertainty principle) combined with the definitions of the Zeeman energy and the magnetic length.

With the Fermi velocity  $v_F$  insensitive to doping [21], Eq. (3) predicts a simple quadratic relation  $H_{c2}^N(p) \propto H_{pg}^2(p)$ . A comparison of  $H_{c2}^N(T \cong 0)$  (see Fig. 2) and  $H_{c2}^*(0)$ , by using most recently measured values for the Fermi velocity [30]  $v_F \simeq 2 \text{ eV}\text{\AA}$  and choosing  $\alpha \approx 0.6$ , yields a proper collapse of the data in the low doping (underdoped) regime  $p < 0.16$ .  $H_{c2}^N(0)$  was obtained in two ways. One, by a simple matching of  $H_{c2}^N$  to  $H_{sc}(0)$  at  $p \gtrsim 0.2$ , where  $H_{sc}(0)$  coincides with the usual  $H_{c2}(0)$ , see Refs. [5,11]. Another, from the ‘universal’  $H_{c2}^N(T)/H_{c2}^N(0)$  vs  $T/T_{ons}^N$  curve implicit in the data of Refs. [17] and [31]; here  $T_{ons}^N$  is the onset temperature of the Nernst response. Close to optimal doping, the scaled and the measured orbital fields part their ways:  $H_{c2}^*$  enters the superconducting ‘dome’ while the  $H_{c2}^N$  follows its edge, pointing to a remarkable distinction between the low- and the high-doping sides [21]. It should be remarked that interlayer tunneling transport is a consistently robust probe of the pseudogap in the underdoped as well as in the overdoped regimes [5], since it measures

electron tunnelling that is sensitive to the spin correlations [27].

Having the two critical fields  $H_{c2}^N$  and  $H_{pg}$  related to a single energy scale  $T^*$ , the question arises how one could dispose of the same correlation energy twice: via the orbital route at  $H_{c2}^N$  and then again via the Zeeman effect at  $H_{pg} \gg H_{c2}^N$ . This ‘double jeopardy’ is naturally resolved by a strongly anisotropic (truncated) Fermi surface [21], hosting separated charge and spin degrees of freedom. A generic starting point is the quantum spin-singlet liquid forming at the energy scale  $T^*$  — this spin-liquid groundstate is void of any long range order and competes with the antiferromagnet [32–34]. Upon doping, the spin-liquid becomes energetically favorable, charge and spin degrees of freedom separate and holes are expected to condense on the spin-liquid background, turning phase coherent at a lower energy  $T_c$ . Recently, ‘cheap’ vortices with staggered-flux cores [33] have been argued to destroy coherence above  $T_c$  and qualitatively explain the Nernst data of Ref. [7], see also [35].

A common feature of these theories is the breakup of the Fermi surface (FS) into regions describing spin-singlet pairs and charged holes: the spin-pairing opens up gaps near the  $(0, \pi)$  points (the FS corners), cf. Fig. 3 — the corresponding pseudogap energy  $T^*$  establishes correlations on the scale  $\xi^* \sim \hbar v_F / T^*$ . Upon doping, a truncated Fermi surface appears around the  $(\pi, \pi)$  diagonals. When charges pair up, they draw correlations from the spin-singlet background, hence spin-singlet pairing at the FS corners and hole-pairing at the diagonals derive from the same energy scale  $T^*$  [34,36], see also [37]. While the pairing energy need not necessarily match the underlying energy scale  $T^*$  of the spin liquid, experiments using scanning tunnelling microscopy (STM) of the vortex cores [35] do show that this is indeed the case in the underdoped cuprates. We emphasize that in the described scenario at sufficiently large doping the separation is ill defined once the spin- and charge degrees of freedom merge into Fermi-liquid type quasiparticles.

The above considerations naturally lead to two field scales  $H_{pg}$  and  $H_{c2}^N$ : the charge degrees are connected to the orbital field  $H_{c2}^N$  obtained from the Nernst transport in Ref. [7]. The *in-plane* Nernst transport reflects the dissipation due to nodal quasiparticles [38] in the vortex cores, with momenta nearly parallel to  $(\pi, \pi)$ . Consequently,  $H_{c2}^N$  inhibits hole-pairing at the FS diagonals, but does not destroy the spin-singlet pairs around the FS corners — these spin-singlets are unpaired at the much higher Zeeman field  $H_{pg}$ . The breakup of the spin-singlets leaves its trace in the *c*-axis tunnelling experiment [39]. Hence, the identification of two limiting magnetic fields  $H_{c2}^N$  and  $H_{pg}$  deriving from the same pseudogap energy scale  $T^*$  via an orbital and a Zeeman relation, respectively, finds a natural interpretation in terms of a reconstructed Fermi surface with separated charge and spin degrees of freedom.

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- [1] U. Welp, W.K. Kwok, G.W. Crabtree, K.G. Vandervoort, and J.Z. Liu, Phys. Rev. Lett. **62**, 1908 (1989).
- [2] K.E. Gray, D.H. Kim, B.W. Veal, G.T. Seidler, T.F. Rosenbaum, and D.E. Farrell, Phys. Rev. B **45**, 10071 (1992).
- [3] G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, and V.M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
- [4] J. Orenstein and A. Millis, Science **288**, 468 (2000).
- [5] T. Shibauchi, L. Krusin-Elbaum, Ming Li, M.P. Maley, and P.H. Kes, Phys. Rev. Lett. **86**, 5763 (2001).
- [6] C. Bernhard, J.L. Tallon, Th. Blasius, A. Golnik, and Ch. Niedermayer, Phys. Rev. Lett. **86**, 1614 (2001).
- [7] Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N.P. Ong, Science **299**, 86 (2003).
- [8] A.P. Mackenzie, S.R. Julian, G.G. Lonzarich, A. Carrington, S.D. Hughes, R.S. Liu, and D.S. Sinclair, Phys. Rev. Lett. **71**, 1238 (1993).
- [9] Y. Ando, G.S. Boebinger, A. Passner, L.F. Schneemeyer, T. Kimura, M. Okuya, S. Watauchi, J. Shimoyama, K. Kishio, K. Tamasaku, N. Ichikawa, and S. Uchida, Phys. Rev. B **60**, 12475 (1999).
- [10] N. Morozov, L. Krusin-Elbaum, T. Shibauchi, L.N. Bulaevskii, M.P. Maley, Yu.I. Latyshev, and T. Yamashita, Phys. Rev. Lett. **84**, 1784 (2000).
- [11] T. Shibauchi, L. Krusin-Elbaum, G. Blatter, and C.H. Mielke, Phys. Rev. B **67**, 64514 (2003).
- [12] A.E. Koshelev, Phys. Rev. Lett. **76**, 1340 (1996).
- [13] M.V. Feigel'man, V.B. Geshkenbein, and A.I. Larkin, Physica C **167**, 177 (1990).
- [14] J.M. Triscone, P. Fivat, M. Andersson, M. Decroux, and Ø. Fischer, Phys. Rev. B **50**, 1229 (1994).
- [15] A.E. Koshelev, private communication (2003).
- [16] H.H. Wen, H.P. Yang, S.L. Li, X.H. Zeng, A.A. Soukiassian, W.D. Si and X.X. Xi, Europhys. Lett. **64**, 790 (2003).
- [17] Y. Wang, N.P. Ong, Z.A. Xu, T. Kakeshita, S. Uchida, D.A. Bonn, R. Liang, and W.N. Hardy, Phys. Rev. Lett. **88**, 257003 (2002).
- [18] C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967).
- [19] M.R. Norman, H. Ding, M. Randeria, J.C. Campuzano, T. Yokoya, T. Takeuchi, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, and D.G. Hinks, Nature **392**, 157 (1998).
- [20] T. Valla, A.V. Fedorov, P.D. Johnson, Q. Li, G.D. Gu, and N. Koshizuka, Phys. Rev. Lett. **85**, 828 (2000).
- [21] A. Damascelli, Z. Hussain, and Z.-X. Shen, Rev. Mod.

- Phys. **75**, 473 (2003).
- [22] M. Suzuki and T. Watanabe, Phys. Rev. Lett. **85**, 4787 (2000); V.M. Krasnov, A.E. Kovalev, A. Yurgens, and D. Winkler, Phys. Rev. Lett. **86**, 2657 (2001).
  - [23] M. Tinkham, *Introduction to Superconductivity*, 2nd ed., McGraw-Hill (New York, 1996).
  - [24] P.A. Lee and X.G. Wen, Phys. Rev. Lett. **78**, 4111 (1997).
  - [25] A.P. Iyengar, Y.-J. Kao, Q. Chen and K. Levin, J. Phys. Chem. Solids **63**, 2349 (2002).
  - [26] R.E. Walstedt, R.F. Bell, L.F. Schneemeyer, J.V. Waszczak, and G.P. Espinosa, Phys. Rev. B **45**, 8074 (1992).
  - [27] L. Krusin-Elbaum, T. Shibauchi, and C.H. Mielke, Phys. Rev. Lett. **92**, 097005 (2004). The orbital anisotropy of the superconducting state is very large.
  - [28] Y.F. Yan, P. Matl, J.M. Harris, and N.P. Ong, Phys. Rev. B **52**, R751 (1995).
  - [29] T. Watanabe, T. Fujii, and A. Matsuda, Phys. Rev. Lett. **84**, 5848 (2000).
  - [30] X.J. Zhou, T. Yoshida, A. Lanzara, P.V. Bogdanov, S.A. Kellar, K.M. Shen, W.L. Yang, F. Ronning, T. Sasagawa, T. Kakeshita, T. Noda, H. Eisaki, S. Uchida, C.T. Lin, F. Zhou, J.W. Xiong, W.X. Ti, Z.X. Zhao, A. Fujimori, Z. Hussain, and Z.-X. Shen, Nature **423**, 398 (2003).
  - [31] Y. Wang, Z.A. Xu, T. Kakeshita, S. Uchida, S. Ono, Y. Ando, and N.P. Ong, Phys. Rev. B **64**, 224519 (2001).
  - [32] P.W. Anderson, Science **235**, 1196 (1987); Science **288**, 480 (2000).
  - [33] Patrick A. Lee, cond-mat/0210113 (2002).
  - [34] C. Honerkamp, M. Salmhofer, N. Furukawa, and T.M. Rice, Phys. Rev. B **63**, 035109 (2001).
  - [35] Ch. Renner, B. Revaz, K. Kadowaki, I. Maggio-Aprile, and Ø. Fischer, Phys. Rev. Lett. **80**, 3606 (1998); the relation  $H_{pg} > H_{c2}^N$  is consistent with pseudogapped vortex cores – observed in STM by Renner *et al.* – moving down a temperature gradient  $-\nabla T$  in the Nernst response [7].
  - [36] C. Honerkamp, M. Salmhofer, and T.M. Rice, Eur. Phys. J. B **27**, 127 (2002). In this weak coupling approach, the Umklapp channel driving the spin-singlet formation and the Cooper channel inducing hole-pairing mutually reinforce one another.
  - [37] V.B. Geshkenbein, L.B. Ioffe, and A.I. Larkin, Phys. Rev. B **55**, 3173 (1997).
  - [38] L. B. Ioffe and A. J. Millis, Phys. Rev. B **58**, 11631 (1998); D. van der Marel, Phys. Rev. B **60**, R765 (1999).
  - [39] K. Takenaka, K. Mizuhashi, H. Takagi, and S. Uchida, Phys. Rev. B **50**, 6534 (1994).

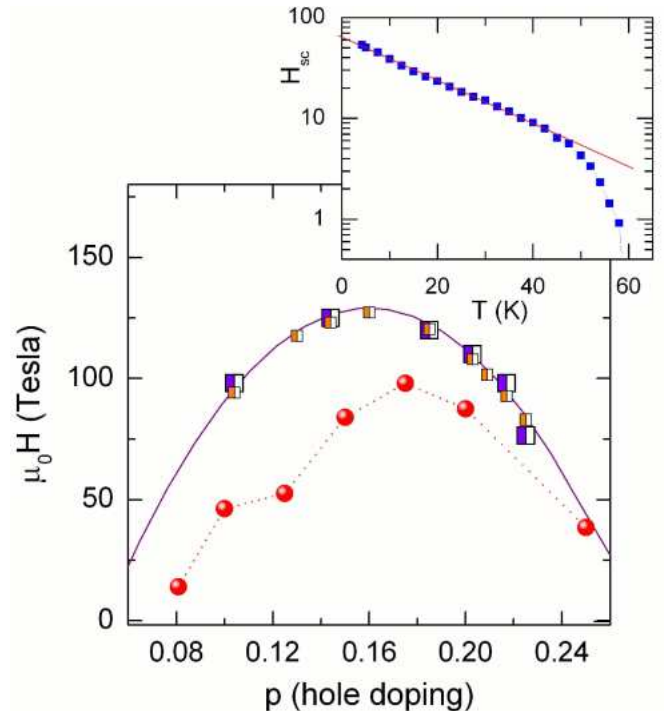


FIG. 1. (Color online) Doping dependence of the peak field  $H_{sc}$  (half-purple squares) in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}$  in the  $T \rightarrow 0$  limit.  $H_{sc}(0, p)$  (in Tesla)  $\sim 1.4T_c$  (in Kelvin, shown as half-orange squares) shapes the ‘superconducting dome’ defined through the presence of large interplane Josephson currents and hosting the ‘conventional’ superconducting phase with recombined quasiparticles. A similar dome-shape is derived from the magnetization measurements of systematically doped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ , shown as red dots (from Ref. [16]). Inset illustrates the nearly  $T$ -exponential temperature dependence of  $H_{sc}(T)$  in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}$  with  $p = 0.225$ , pointing to the  $T = 0$  value of  $H_{sc}$ . See also Ref. [5].

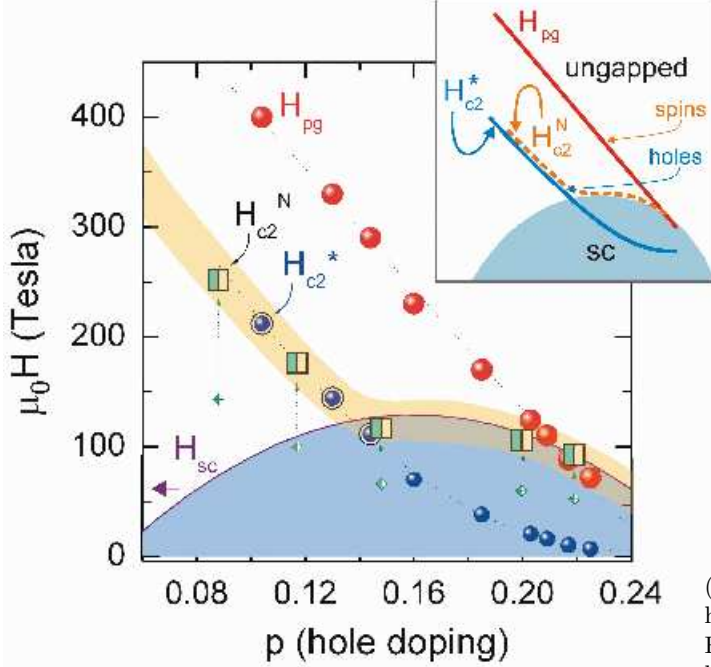


FIG. 2. (Color online) Comparison of  $H_{pg}$ ,  $H_{c2}^N$  in  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}$ , and the transformation from  $H_{pg}$  to  $H_{c2}^*$  via Eq. (3), where we used [30]  $v_F \approx 1.8 \pm 0.2 \text{ eV\AA}$  and  $\alpha \approx 0.6$ ; note the single ' $H_{c2}$ ' scale in the low doping regime. As sketched for emphasis in the inset, the lines  $H_{c2}^N(0)$  and  $H_{c2}^*(0)$  split apart upon reaching the superconducting dome (shaded sky-blue)  $H_{sc}(0, p)$ .  $H_{c2}^N$  data (half-green diamonds) is from scaling near  $T_c$  (Ref. [7]). Light yellow band follows  $H_{c2}^N(0)$  (half-green squares) obtained as described in the text.

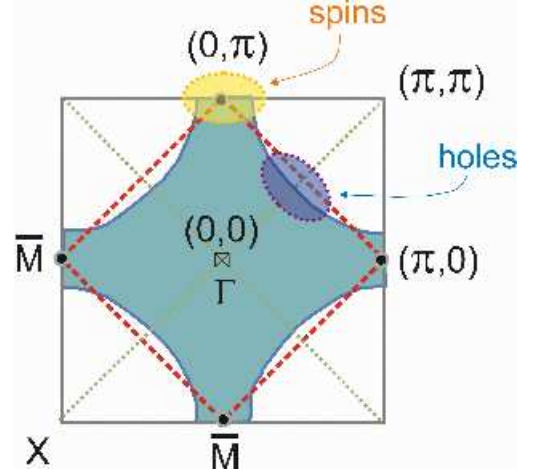


FIG. 3. (Color online) The breakup of the Fermi surface (FS) into regions describing spin-singlet pairs and charged holes: the spin-pairing opens up gaps at the  $(0, \pi)$  points (the FS corners) — the corresponding pseudogap energy  $T^*$  establishes correlations on the scale  $\xi^* \sim \hbar v_F / T^*$ . Upon doping, a truncated FS appears around the  $(\pi, \pi)$  diagonals. When charges pair up, they draw correlations from the spin-singlet background, hence spin-singlet pairing at the FS corners and hole-pairing at the diagonals derive from the same energy scale  $T^*$ . The separation of spins and holes will eventually become 'fuzzy' in the overdoped regime and there may be an overlap on the Fermi surface. In this limit the two (in-plane and out-of-plane) experimental transport probes will not detect the differences.